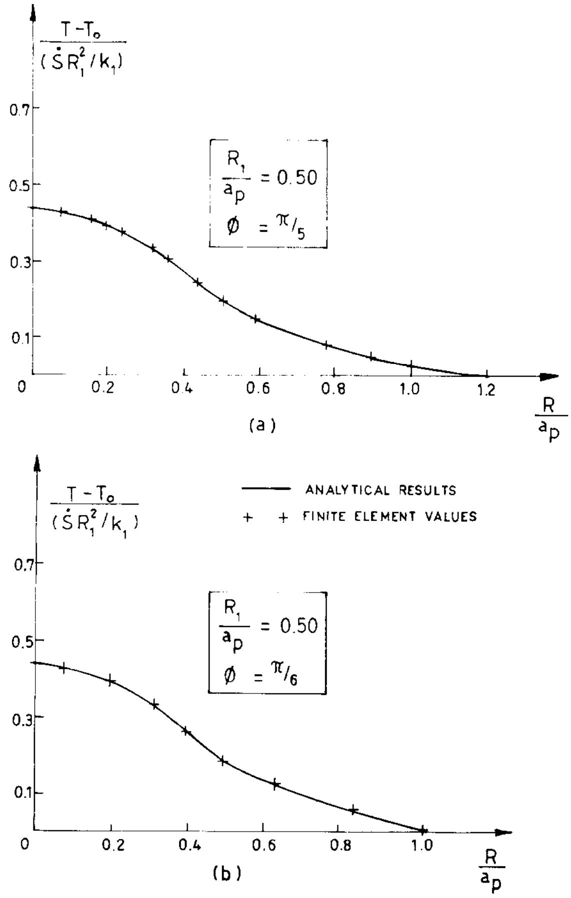

FIG. 2. Dimensionless temperature distribution. (a) Pentagonal outer boundary. (b) Hexagonal outer boundary.

and from (11) and (17) one finally obtains:

$$\frac{T-T_0}{\frac{\dot{S}\cdot R_1^2}{k_1}} = -\frac{\ln r}{2(k_2/k_1)}. \tag{18}$$

Using now equations (7) and (18) one determines the thermal field in the sub-domain II.

Numerical results of the problem governed by equations (1), (2) and (3) were also obtained using a finite element code.* The domain is subdivided into triangular elements and a linear variation of the temperature field is assumed inside the element [see Fig. 1(c)].

The accuracy of the algorithm is quite satisfactory from a practical viewpoint since it yields an agreement better than 0.5% with exact solutions when using a subdivision similar to that shown in Fig. 1(c).

COMPARISON OF RESULTS AND CONCLUSIONS

Table 1 depicts a comparison of results for an outer square shape ($R_1/R_p = 0.20$ and 0.50; $\phi = 0°$).

Figure 2 deals with pentagonal and hexagonal boundaries and the dimensionless temperature parameter has been plotted for $\phi = \pi/5$ and $\pi/6$, respectively.

All calculations have been performed taking $k_2/k_1 = 2$.

It may be concluded that the agreement is remarkably good for the cases considered and even in the cases where the values of R_1/a_p are relatively large (in spite of the approximations involved when using the analytical approach).

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*Developed at Centro Atómico Bariloche, CNEA.

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NATURAL CONVECTION FROM SINGLE HORIZONTAL PLATINUM WIRES

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NOMENCLATURE

C_p, fluid heat capacity at constant pressure;
D, wire diameter;
g, acceleration of gravity;
Gr, Grashof number, $l^3\rho^2 g\beta\theta/\eta^2$;
h, heat-transfer coefficient;
l, wire length;
Nu, Nusselt number, hD/λ;
Pr, Prandtl number, $C_p\eta/\lambda$;
T, temperature;
T_0, temperature of the enclosure.

Greek symbols

β, fluid coefficient of thermal expansion;
η, dynamic viscosity;
θ, wire to wall temperature difference;
λ, fluid thermal conductivity;
ρ, fluid density.

INTRODUCTION

CONVECTIVE heat transfer is governed by the laws of fluid flow, the equation of continuity, and the equation for the heat flow in a moving fluid. An exact solution of these equations with particular boundary conditions is not feasible except in certain simple cases. However, important relationships may be obtained from these equations by means of the theory of similarity. Thus for a natural convection the Nusselt number should be a function of the Grashof number and the Prandtl number, i.e.

$$Nu = f(Gr, Pr). \tag{1}$$

The form of $f(Gr, Pr)$ can be determined either strictly experimentally or by using theoretical analysis with some experimental information.

Our current understanding of convective heat transfer from circular cylinders has been summarized in the recent review by Morgan [1]. According to his extensive survey there is a wide dispersion in the published experimental data on natural convection from horizontal circular cylinders. These inconsistencies can be attributed to one or more of the following factors: heat conduction to the supports and the temperature measurement locations, distortion of the temperature and velocity fields by bulk fluid movements, the use of undersized containing chambers, and temperature loading effects. The axial conduction losses have been particularly troublesome in most experimental studies. From the work by Collis and Williams [2, 3], Gosse [4], Gebhart and Pera [5], and Gebhart, Anderson and Pera [6] using fine wires in air it is clear that in hot-wire measurements the end losses can be neglected if the aspect ratio l/D is larger than 10^5. This condition is difficult to satisfy. Very few measurements have been made with $l/D > 10^4$. For smaller values of l/D, corrections for end heat losses have to be applied. Usually, this has not been done. In fact, many previous studies do not give sufficient information for making such corrections. This observation stimulated us to explore the experimental behavior of Nu as a function of $GrPr$ between 10^{-5} and 10^{-2} using data which one of the authors obtained some time ago [7]. In this report we present the experimental values of Nu obtained for various gases as a function of $GrPr$ and compare them with some recently proposed formulas for $Nu = f(Gr, Pr)$.

EXPERIMENTAL CONSIDERATION

The general description of the equipment used for the heat-transfer studies has been given elsewhere [7, 8]. Four hot wire cells were used. Cells 1 and 2 were made from copper cylinders with fine platinum wires [diameter $(5.08 \pm 0.09) \times 10^{-3}$ cm, length 8.65 ± 0.05 and 8.83 ± 0.05 cm, respectively] stretched along the axis of each cylinder (Fig. 3, of [7]). The diameter of cells 1 and 2 were 5.43 and 2.45 cm, respectively. In cells 3 and 4 platinum wires [diameter $(7.62 \pm 0.09) \times 10^{-3}$ cm, length 13.44 ± 0.05 cm and diameter $(5.08 \pm 0.09) \times 10^{-3}$ cm, length 13.41 ± 0.05 cm, respectively] were mounted in a large copper container (inside diameter 19 cm, height 25 cm) as shown in Fig. 4 of [7].

The source of various transport and thermal properties needed for the evaluation of the dimensionless numbers was the same as that used in the electroconvection studies [8]. The properties appearing in the characteristic numbers were evaluated at $T = T_0 + \frac{1}{2}\theta$.

RESULTS AND DISCUSSION

Figure 1 shows an experimental value of Nu as a function of $GrPr$. The values of Nu have been corrected for the heat losses due to finite length of the heated wires. The apparent increase ΔNu was calculated using the empirical equation (1)

$$\Delta Nu = \frac{13.3 Nu}{(l/D)^{0.566}} \tag{2}$$

which satisfies the experimental data of [2–6]. According to equation (2) the quantity $13.3/(l/D)^{0.566}$ for cells 1–3 equals 0.20 while that for cell 4 is about 0.15. Our experimental points for various gases and cell geometries fall on a dotted curve labelled as Kyte *et al.*—experimental. This curve, which smoothly extends down to $GrPr = 10^{-7}$, was obtained from Fig. 17, of [9] after necessary corrections. The data given in this figure were corrected by us for end losses using equation (2) and the aspect ratio of 1910, i.e. the correction factor $13.3/(l/D)^{0.566}$ was taken to be 0.18. It is obvious from Fig. 1 that the corrected experimental data by Kyte *et al.* [9] and our data agree satisfactorily in the region of overlapping $GrPr$. Figure 1 also shows the proposed empirical curve by Morgan [1]. Specifically, Morgan, after a careful analysis of all published experimental data together with possible errors, proposed that

$$Nu = C(GrPr)^m, \tag{3}$$

where $C = 0.675$ and $m = 0.058$ for $GrPr$ from 10^{-10} to 10^{-2}. It is suggested that the proposed correlation has a maximum uncertainty of $\pm 5\%$. Morgan's curve with indicated $\pm 5\%$ uncertainties lies slightly above the experimental data of our studies and corrected values due to Kyte *et al.* The reason for this discrepancy is not clear at the present time. New and careful experimental studies using wires of aspect ratios in the neighborhood of 10^5 should help to clarify this situation.

Figure 1 also presents some other proposed correlations of equation (1). The curve due to Kyte *et al.* [9] is a plot of

$$Nu = \left\{ \ln\left[1 + \frac{7.09}{(GrPr)^{0.37}}\right] \right\}^{-1}. \tag{4}$$

Recently Churchill and Chu [10] have published the following semi-empirical correlation equation for heat transfer by natural convection from horizontal cylinders:

$$Nu = 0.36 + 0.518 \left\{ \frac{GrPr}{[1 + (0.559/Pr)^{9/16}]^{16/9}} \right\}^{0.25}. \tag{5}$$

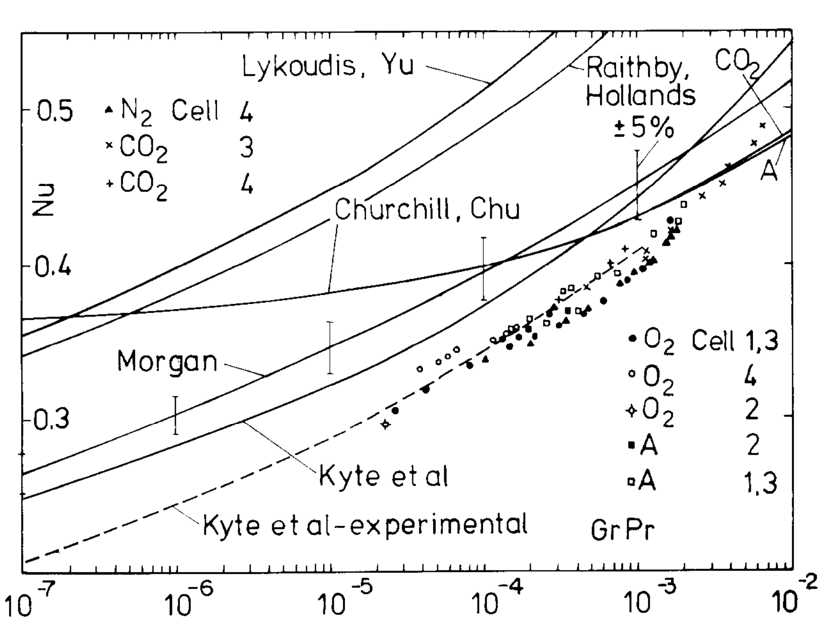

FIG. 1. Experimental $Nu = f(GrPr)$ in comparison with various proposed correlations.

Churchill and Chu state in their publication that this equation provides a good fit of representative data for all Pr and $10^{-6} \leqslant GrPr \leqslant 10^9$, except for the experimental data of Collis and Williams [2], which fall below equation (5) for $GrPr \leqslant 10^{-6}$. We have calculated the values of Nu according to equation (5) for carbon dioxide and argon as a function $GrPr$. These curves are shown in Fig. 1. Both curves coincide except in the region of $GrPr$ between 10^{-3} and 10^{-2} where the curve for argon lies very slightly below that of carbon dioxide. It is clear from Fig. 1 that the correlation represented by equation (5) does not agree with our data. It should be remarked that in their paper Churchill and Chu are plotting $\log Nu$ vs $\log GrPr/[1+(0.559/Pr)^{9/16}]^{16/9}$. Our Fig. 1 shows Nu on a linear scale which is much more sensitive to small but still experimentally detectable deviations in the quantity Nu.

Another recent correlation equation for horizontal cylinders is due to Raithby and Hollands [10]:

$$Nu = \left\{\frac{2}{\ln[1+\pi 2^{3/4}/2.040A(GrPr)^{1/4}]}\right\}^{3.337} + [0.72B(GrPr)^{1/3}]^{3.337}, \quad (6)$$

where

$A = (2/3)/[1+(0.49/Pr)^{9/16}]^{4/9}$

$B = 0.14Pr^{0.084}$ or 0.15, whichever is smaller.

This function, calculated using the value of Pr corresponding to oxygen, is plotted in Fig. 1. It predicts values for given Gr and Pr data which are too large in comparison with the experimental data for $GrPr$ between 10^{-7} and 10^{-2}.

Lastly, our Fig. 1 presents the correlation curve used by Lykoudis and Yu [11] in their theory of electroconvectional heat transfer from horizontal cylinders:

$$Nu = 2\{\ln[1+(546/GrPr)^{1/4}]\}^{-1}. \quad (7)$$

This curve lies slightly above that of Raithby and Hollands and thus overestimates the natural convectional heat loss from a horizontal cylinder.

In summary, the above analysis reveals two observations. First, the recently proposed correlations for equation (1) do not correctly represent the experimental data for small $GrPr$ values. Second, new experimental data obtained with negligible end losses from heated wires should help to clarify the correct heat-transfer characteristics from horizontal cylinders in the region where $GrPr < 10^{-3}$.

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